

LGIC 320 / MATH 571: Logic II

Preliminary Outline

1. Propositional intuitionistic logic (Int)

- Foundations and motivations. BHK semantics
- Hilbert-style axiomatization
- Kripke semantics, completeness theorem
- Disjunctive property; double-negation translation ($CL \rightarrow Int$)
- Finite-valued logics; Int is not finite-valued
- Topological semantics; completeness theorem

2. Lambda calculus

- Untyped lambda calculus as a universal computational model
- The Church – Rosser (diamond) property
- Simply typed lambda calculus
- Strong normalization [proof optional]
- Set-theoretic models for lambda-calculus, completeness theorem
- Natural deduction for Int; the Curry – Howard correspondence
- Combinators and the Hilbert-style calculus for Int

3. First-order intuitionistic logic (FO-Int)

- Hilbert-style axiomatization
- Kripke semantics, completeness theorem
- Double-negation translation ($FO-CL \rightarrow FO-Int$)
- Disjunctive property; constructivity of the existential quantifier
- The constant domain principle
- Curry – Howard for FO-Int; calculus of inductive constructions (CIC)
- Application: the Coq proof assistant

4. Sequent (Gentzen-style) calculi

- Sequent calculi for FO-CL and FO-Int
- Cut elimination: semantic and syntactic approaches
- Disjunctive property, constructivity of the existential quantifier, and Herbrand's theorem syntactically
- Substructural logics, linear logic and its variants
- Semi-Thue systems; undecidability of propositional linear logic
- Application: non-commutative linear logic in linguistics (the Lambek calculus)

5. Modal logic

- Kripke semantics for modal logic, complete and incomplete logics
- Canonical model, canonicity
- Sahlqvist formulae, Sahlqvist's completeness theorem [proof optional]
- Completeness and finite model property of GL (Gödel-Löb logic)
- Topological semantics for S4 and GL
- Arithmetical interpretation of GL, Solovay's theorem [proof optional]
- Sequent calculi for modal logics; infinite and cyclic proofs for GL