Lambek Categorial Grammars
Day 5

Stepan Kuznetsov

Steklov Mathematical Institute, RAS
for ESSLLI '15 in Barcelona

August 7, 2015

Context-Free Languages, Homomorphisms, and Regular Intersections

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If $M \subseteq \Sigma^*$ is context-free and $R \subseteq \Sigma^*$ is regular, the $M \cap R$ is context-free.

These transformations are polynomial.

Lemma

If a language M is generated by a categorial grammar $\mathcal{G}$, then there exists a categorial grammar $\mathcal{G}'$ with unique type assignment, that generates the language M', and a homomorphism h, such that $M = h(M')$.

The Lambek Calculus with One Division

$$A \to A$$

$$\frac{A\,\Pi \to B}{\Pi \to A \setminus B}\text{, } \Pi \text{ is not empty} \qquad \frac{\Pi \to A \quad \Gamma\,B\,\Delta \to C}{\Gamma\,\Pi\,(A \setminus B)\,\Delta \to C}$$

$$\frac{\Pi \to A \quad \Gamma\,A\,\Delta \to C}{\Gamma\,\Pi\,\Delta \to C}\ (\mathrm{cut})$$

Savateev's Proof Nets

$\mathrm{Atn} = \{p^{(i)} \mid p \in \mathrm{Pr},\ i \in \mathbb{N}\}.$

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$\gamma, \bar{\gamma} \colon \mathrm{Tp}(\backslash) \to \mathrm{Atn}^{+}$.

$$\gamma(p) = p^{(1)} \qquad \bar{\gamma}(p) = p^{(2)}$$
$$\gamma(A \setminus B) = \bar{\gamma}(A)\gamma(B) \qquad \bar{\gamma}(A \setminus B) = \bar{\gamma}(B)\big(\gamma(A)\big)^{+2}$$

Translate $A_1 \ldots A_n \to B$ into $\gamma(A_1) \ldots \gamma(A_n)\bar{\gamma}(B)$.

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Example. $(r \setminus p)\ ((s \setminus p) \setminus t) \to (s \setminus r) \setminus t$.

$r^{(2)} p^{(1)} p^{(2)} s^{(3)} t^{(1)} t^{(2)} s^{(4)} r^{(3)}$

Savateev's Proof Nets

A sequent is derivable in $\mathrm{L}(\backslash)$ iff on its translation to Atn^+ there exists such pairing of letters, that

- a pair consists of $p^{(i)}$ and $p^{(i+1)}$, and $p^{(i)}$ stays to the left;
- the links connecting pairs can be drawn in the upper semiplane without intersections;
- if the superscript of the left atom in the pair is even, then there is an atom with a less superscript between the elements of the pair.

Yury Savateev

Proof Net Conditions as a Context-Free Grammar

Let $\Sigma_1 = \mathrm{Atn}_m$. Then there exists a context-free grammar generating the language

$M_1 = \{\mathbf{a}_1 \ldots \mathbf{a}_n \in \mathrm{Atn}_m^+ \mid$
this string satisfies the conditions from the previous slide$\}$.

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Rules:

$S \to S_k$ for every k
$S_k \to p^{(2\ell-1)} S_k p^{(2\ell)}$
$S_{2\ell-1} \to p^{(2\ell-1)} S p^{(2\ell)}$
$S_{2\ell-1} \to p^{(2\ell-1)} p^{(2\ell)}$

$S_{k_1} \to S_{k_2}$, if $k_1 > k_2$
$S_k \to p^{(2\ell)} S_k p^{(2\ell+1)}$, if $k < 2\ell$
$S_k \to S_k S$, $S_k \to S S_k$

Two Homomorphisms and One Regular Language

Let grammar $\mathcal{G}$ define the language $M \subseteq \Sigma^+$. Then there exists a grammar $\mathcal{G}_2$ with unique type assignment, and $M = h(M_2)$ ($h\colon \Sigma_2 \to \Sigma$ is a homomorphism).

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Now define a homomorphism $g \colon \Sigma_2 \cup \{\$\} \to \Sigma_1$:
$g(a) = \gamma(A)$, where $\langle A, a \rangle \in \mathcal{D}_2$ and $g(\$) = \bar{\gamma}(H)$.

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Finally, $M = h(g^{-1}(M_1) \cap R)$.

Medial Extraction and Islands

John read *Ulysses* a month ago.

the book which John read [] a month ago

Medial Extraction and Islands

John read *Ulysses* a month ago.
the book which John read [] a month ago

John sings and loves Mary.
*the girl which John sings and loves

Medial Extraction: First Moortgat's Approach

$$\frac{\Gamma, x : A, \Delta \to u : B}{\Gamma, \Delta \to \lambda x.u : B \uparrow A}$$

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No left rule (unfortunately).

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$$\frac{\Gamma, x : A, \Delta \rightarrow u : B}{\Gamma, \Delta \rightarrow \lambda x.u : B \uparrow A}$$

No left rule (unfortunately). Could lead to the displacement calculus.

"Bracket" Modalities

$$A \rightarrow A$$

$$\frac{\Gamma \rightarrow A \quad \Delta(A) \rightarrow C}{\Delta(\Gamma, A \backslash B) \rightarrow C} \qquad \frac{A, \Gamma \rightarrow B}{\Gamma \rightarrow A \backslash B}$$

$$\frac{\Gamma \rightarrow A \quad \Delta(A) \rightarrow C}{\Delta(B / A, \Gamma) \rightarrow C} \qquad \frac{\Gamma, A \rightarrow B}{\Gamma \rightarrow B / A}$$

$$\frac{\Delta([A]) \rightarrow B}{\Delta(\Diamond A) \rightarrow B} \qquad \frac{\Gamma \rightarrow A}{[\Gamma] \rightarrow \Diamond A}$$

$$\frac{\Delta(A) \rightarrow B}{\Delta([\Box^{-1} A]) \rightarrow B} \qquad \frac{[\Gamma] \rightarrow A}{\Gamma \rightarrow \Box^{-1} A}$$

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Residuation pair: $\Diamond\Box^{-1}A \to A \to \Box^{-1}\Diamond A$.

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Morrill 1992, Moortgat 1995, Fadda & Morrill 2005

Glyn Morrill & Michael Moortgat with Raffaella Bernardi

Ad-Hoc Nonassociativity

Bracket modalities locally kill associativity.

cf. Kurtonina '95: embedding of non-associative Lambek calculus into Lambek calculus with bracket modalities:

$A \backslash_{\mathrm{NA}} B = A \backslash \Box^{-1} B$, $B /_{\mathrm{NA}} A = \Box^{-1} B / A$, $A \cdot_{\mathrm{NA}} B = \Diamond (A \cdot B)$.

Brackets Forbid Extraction (Create Islands)

John likes Mary and Mary likes Pete.
*man that John likes Mary and Mary likes []

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John	likes	Mary	and	Mary	likes	Pete
np	$(np \backslash s) / s$	np	$(s \backslash \Box^{-1} s) / s$	np	$(np \backslash s) / s$	np

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$$\dfrac{\dfrac{\dfrac{s \to s}{[\Box^{-1} s] \to s}}{\cdots}}{[np, (np \backslash s) / s, np, (s \backslash \Box^{-1} s) / s, np, (np \backslash s) / s, np] \to s}$$

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man	that	John	likes	Mary	and	Mary	likes
n	$(n \backslash n) / (s / np)$	np	$(np \backslash s) / np$	np	$(s \backslash \Box^{-1} s) / s$	np	$(np \backslash s) / np$

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man	that	[John	likes	Mary	and	Mary	likes]
n	$(n \backslash n)/(s / np)$	np	$(np \backslash s) / np$	np	$(s \backslash \Box^{-1} s) / s$	np	$(np \backslash s) / np$

$$\dfrac{\dfrac{\dfrac{[s, (s \backslash \Box^{-1} s) / s, np, (np \backslash s) / np], np \to s}{[s, (s \backslash \Box^{-1} s) / s, np, (np \backslash s) / np] \to s / np}}{\cdots}}{[np, (np \backslash s) / np, np, (s \backslash \Box^{-1} s) / s, np, (np \backslash s) / np] \to s / np}$$

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$$\dfrac{\dfrac{np, np \backslash s, ((np \backslash s) \backslash (np \backslash s)) \to s \quad [(np \backslash s') / np], \color{red}{np} \to \Diamond(np \backslash s')}{np, np \backslash s, ((np \backslash s) \backslash (np \backslash s)) / \Diamond(np \backslash s'), [(np \backslash s') / np], \color{red}{np} \to s}}{np, np \backslash s, ((np \backslash s) \backslash (np \backslash s)) / \Diamond(np \backslash s'), [(np \backslash s') / np] \to s / np}$$

Medial Extraction

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$$\frac{\Gamma \to A \quad \Delta(A) \to C}{\Delta(\Gamma, A \backslash B) \to C} \quad \frac{A, \Gamma \to B}{\Gamma \to A \backslash B} \quad \frac{\Delta([A]) \to B}{\Delta(\Diamond A) \to B} \quad \frac{\Gamma \to A}{[\Gamma] \to \Diamond A}$$

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$$\frac{\Delta(\{A\}) \to B}{\Delta(\blacklozenge A) \to B} \qquad \frac{\Gamma \to A}{\{\Gamma\} \to \blacklozenge A}$$

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Medial Extraction

man	who	John	saw	yesterday
n	$(n \backslash n) / (s / \blacklozenge \blacksquare^{-1} np)$	np	$(np \backslash s) / np$	$(np \backslash s) \backslash (np \backslash s)$

Medial Extraction

man	who	John	saw	yesterday
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Medial Extraction

man	who	John	saw	yesterday
n	$(n \backslash n) / (s / \blacklozenge\blacksquare^{-1} np)$	np	$(np \backslash s) / np$	$(np \backslash s) \backslash (np \backslash s)$

$$\dfrac{\dfrac{\dfrac{\dfrac{np, (np \backslash s) / np, np, (np \backslash s) \backslash (np \backslash s) \rightarrow s}{np, (np \backslash s) / np, \{\blacksquare^{-1} np\}, (np \backslash s) \backslash (np \backslash s) \rightarrow s}}{np, (np \backslash s) / np, (np \backslash s) \backslash (np \backslash s), \{\blacksquare^{-1} np\} \rightarrow s}}{np, (np \backslash s) / np, (np \backslash s) \backslash (np \backslash s), \blacklozenge\blacksquare^{-1} np \rightarrow s}}{np, (np \backslash s) / np, (np \backslash s) \backslash (np \backslash s) \rightarrow s / \blacklozenge\blacksquare^{-1} np}$$

$B / \blacklozenge\blacksquare^{-1} A$ works as $B \uparrow A$.

Further Reference

G. Morrill. Categorial grammar. Logical syntax, semantics, and processing

Thank you for following this course

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