

Birational Geometry and Fano varieties
Steklov Mathematical Institute, June 24-28, 2019

Abstracts

Valery Alexeev

Stable pair compactifications of the moduli of elliptic K3 surfaces

I will describe two compactifications of moduli of K3 surfaces via stable slc pairs given by two different choices of a polarizing divisor. This is a joint work with Philip Engel.

Florin Ambro

Successive minima of line bundles

I will discuss a connection between Seshadri constants of line bundles on projective varieties, and successive minima of a 0-symmetric convex body with respect to a lattice. We introduce the successive minima of a line bundle on a projective variety, such that the last minima is the Seshadri constant. We show the analogue of Minkowski's second theorem, namely that the volume of a line bundle is proportional with the product of its successive minima at a very general point. Based on joint work with Atsushi Ito.

Victor Batyrev

Minimal models of surfaces with $p_g = 1, q = 0$ associated with canonical Fano 3-polytopes

Let Δ be a canonical Fano 3-polytope, i.e., a 3-dimensional lattice polytope containing exactly one interior lattice point. Then the affine surface Z_Δ defined by a generic Laurent polynomial f_Δ with the Newton polytope Δ is birational to a smooth projective minimal surface S_Δ with $q = 0$ and $p_g = 1$. Using the classification of all 674,688 canonical Fano 3-polytopes obtained by Kasprzyk, we show that S_Δ is a K3-surface except for exactly 9,089 canonical Fano 3-polytopes Δ . In the latter case, we obtain 9,040 canonical Fano 3-polytopes Δ defining minimal elliptic surfaces S_Δ of Kodaira dimension 1 and 49 canonical Fano 3-polytopes Δ defining minimal surfaces S_Δ of general type with $|\pi_1(S_\Delta)| = K^2 \in \{1, 2\}$ considered by Kynev and Todorov. This is a joint work with Kasprzyk and Schaller.

Arnaud Beauville

Vector bundles on Fano threefolds and K3 surfaces

Let X be a Fano threefold, and let $S \subset X$ be a smooth anticanonical surface (hence a K3). Any moduli space \mathcal{M}_S of simple vector bundles on S carries a holomorphic symplectic structure. Following an idea of Tyurin, I will show that in some cases, those vector bundles which come from X form a Lagrangian subvariety of \mathcal{M}_S . Most of the talk will be devoted to concrete examples of this situation.

Jérémy Blanc

Quotients of higher dimensional Cremona groups

We study large groups of birational transformations $\text{Bir}(X)$, where X is a variety of dimension at least 3, defined over \mathbb{C} or a subfield of \mathbb{C} . Two prominent cases are when X is the projective space \mathbb{P}^n , in which case $\text{Bir}(X)$ is the Cremona group of rank n , or when $X \subset \mathbb{P}^{n+1}$ is a smooth cubic hypersurface. In both cases, and more generally when X is birational to a conic bundle, we produce infinitely many distinct group homomorphisms from $\text{Bir}(X)$ to $\mathbb{Z}/2$. As a consequence we also obtain that the Cremona group of rank $n \geq 3$ is not generated by linear and Jonquieres elements. Joint work with Stéphane Lamy and Susanna Zimmermann.

Fedor Bogomolov

Group theoretic prove of the classification of VII_0 surfaces with $b_2 = 0$

In this talk I will discuss our joint work with Nikon Kurnosov and Federico Buonerba where we simplified my initial proof for the classification of such surfaces. We reduce almost all geometric elements in the initial proof to group theoretic arguments using the results on the structure of groups acting effectively on trees

Kento Fujita

Toward criteria for K-stability of log Fano pairs

We survey recent progresses related to “a valuative criterion for K-stability of log Fano pairs”. If we have any time left, I will especially focus on K-stability of asymptotically log del Pezzo surfaces motivated by the conjecture of Cheltsov and Rubinstein.

Yoshinori Gongyo

Nef anti-canonical divisors and rationally connected fibrations

We study the Iitaka-Kodaira dimension of nef relative anti-canonical divisors. As a consequence, we prove that given a complex projective variety with klt singularities, if the anti-canonical divisor is nef, then the dimension of a general fibre of the maximal rationally connected

fibration is at least the Iitaka-Kodaira dimension of the anti-canonical divisors. This is a joint work with Sho Ejiri.

Christopher Hacon

On the minimal model program for 3-folds in small characteristic

In this talk I will discuss recent progress on the birational geometry of algebraic varieties of dimension 3 over an algebraically closed field of characteristic $p = 2, 3, 5$. In particular I will discuss the minimal model program for threefolds in characteristic 5 and the birational minimal model program for 3-folds in characteristics 2, 3. This is joint work in progress with J. Witaszek

Yujiro Kawamata

On non-commutative deformations and applications to perverse coherent sheaves and semi-orthogonal decompositions

We consider deformations of objects over non-commutative rings, which is more natural than commutative ones in some cases. We consider examples, and applications to perverse coherent sheaves, and semi-orthogonal decompositions of derived categories corresponding to singularities.

Ludmil Katzarkov

Birational and symplectic invariants from HMS

We will introduce some new birational and symplectic invariants. Examples will be considered - remaining issues will be discussed. This is a joint work with M. Kontsevich and T. Pantev.

Keiji Ogusio

Coble's question and complex dynamics of inertia groups on K3 surfaces

We study the inertia groups of some smooth rational curves on 2-elementary K3 surfaces and singular K3 surfaces from the view of topological entropy, with an application to a long standing open question of Coble on the inertia group of a generic Coble surface. This is a joint work with Xun Yu.

Laurent Manivel

On Fano symmetric varieties

Takuzo Okada

Birationally superrigid Fano 3-folds of codimension 4

We have satisfactory results on both classification and birational (super)rigidity for (terminal) Fano 3-folds of index 1 embedded in a

weighted projective space as a codimension at most 3 subvariety. I will talk about birational superrigidity of Fano 3-folds of index 1 embedded in a wps as a codimension 4 subvariety.

Jihun Park

Sasaki-Einstein metrics on simply connected rational homology 5-spheres

By developing the method introduced by Kobayashi in 1960's, Boyer, Galicki and Kollár found many examples of simply connected Sasaki-Einstein 5-manifolds. For such examples they verified existence of orbifold Kähler-Einstein metrics on various log del Pezzo surfaces. Their verification is essentially based on the α -invariant method developed by Tian, Nadel, Demailly and Kollár. Using the δ -invariant instead of the α -invariant on the leaf spaces of the Reeb foliations of Sasakian 5-manifolds, I complete the classification of simply connected rational homology 5-spheres that admits Sasaki-Einstein metrics. This is a joint work with Joonyeong Won.

Miles Reid

The Tate-Oort group scheme TO_p

Godeaux surfaces over \mathbb{C} are constructed in terms of μ_5 -equivariant quintic hypersurfaces in \mathbb{P}^3 , giving a quotient surface X having $\mathbb{Z}/5$ in $\mathrm{Pic} X$. The Tate-Oort group scheme TO_p answers the same kind of questions in characteristic p and in mixed characteristic at p . (I lectured on this at HSE in November 2017.) My lecture will give an overview of theoretical points in the construction of TO_p , its Cartier dual $(\mathrm{TO}_p)^\vee$, its representation theory and many types of applications.

For more details and the current draft of a preprint, see my website + TO_p .

Artan Sheshmani

Lagrangian distributions, shifted symplectic structures and higher dimensional gauge theory on Calabi-Yau 4-folds

We discuss a generalized construction of a compact moduli space of spin 7 instantons, or coherent sheaves on Calabi Yau 4 folds, as a global derived critical locus of a graded potential. We discuss a generalization to Joyce's local construction of this moduli space as a derived critical locus. If time permits we report on some applications of this construction. This is report on joint projects with Borisov-Katzarkov, and Borisov-Yau.

Vyacheslav Shokurov

Application of complements

In the last decade unexpectedly, some expected properties of thresholds related to singularities and intersection properties were established. In its turn, the acc for lc thresholds implies the uniform stability of lc singularities and of the \mathbb{R} -complementary property. We present a different approach to these and other results related to thresholds based on the theory of complements. We start from an inversion of both uniform stabilities. After that we explained how the inversion can be applied to obtained some of already known and new results, including but not only, lc and Fano index thresholds, lc indices, a , α -invariants.

Constantin Shramov

Finite groups of birational selfmaps

Groups of birational selfmaps of algebraic varieties may be large and difficult to understand. On the other hand, the structure of their finite subgroups is often much more accessible. One example is given by K3 surfaces, whose automorphism group may be infinite but always contains just a finite number of finite subgroups up to isomorphism. I will survey the results on boundedness of finite groups acting by automorphisms and birational automorphisms of algebraic varieties, and discuss other examples with similar properties.

Claire Voisin

Surface decompositions of hyper-Kähler manifolds

In recent years, new constructions of complete families of polarized hyper-Kähler manifolds have been found starting from Fano geometry. These hyper-Kähler manifolds also appear as deformations of Hilbert schemes of K3 surfaces or O'Grady manifolds. I will introduce the notion of surface decomposition, which is restrictive and gives a geometric explanation for the Beauville-Fujiki relations, and I will show the existence of such a surface decomposition for the general hyper-Kähler manifold mentioned above. This has implications on the Beauville conjecture.

Jaroslav Wisniewski

Small bandwidth \mathbb{C}^ -actions and birational geometry*

Dealing with Oda and Hironaka conjectures about factorization of birational maps, in 1990's Morelli and Włodarczyk showed that birational maps can be related to varieties with \mathbb{C}^* -action via algebraic

cobordisms construction. We note that conversely, varieties with \mathbb{C}^* -action yield interesting birational transformations. This topic is a part of a joint project with Occhetta, Romano, and Sola Conde about varieties with contact structure.