WORKSHOP ON BIRATIONAL GEOMETRY
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ABSTRACTS

Mathieu Florence (Université Paris VI)
Rationality questions for forms of moduli spaces, I

This talk is about joint work with Zinovy Reichstein. Let F be a field. Denote by $\mathcal{M}_{0,n}$ the moduli space of (say, smooth) curves of genus zero, equipped with $n \geqslant 4$ distinct marked points. As an F-variety, it is F-rational, of dimension $n-3$. One can consider its twisted forms, that is to say, the F-varieties X, becoming isomorphic to $\mathcal{M}_{0,n}$ over an algebraic closure of F. Our main theorem then goes a follows. If $n \geqslant 5$ is odd, then all F-forms of $\mathcal{M}_{0,n}$ are F-rational. If $n \geqslant 6$ is even, then there exists (over some F) non F-retract rational such F-forms. Note that F-forms of $\mathcal{M}_{0,5}$ are exactly del Pezzo surfaces of degree 5. Hence, the positive part of our theorem generalizes a well-known result of Manin and Swinnerton-Dyer, asserting that Del Pezzo surfaces of degree 5 are rational. I shall discuss the various techniques involved in the proof, strongly related to Noether's problem.

Mathieu Florence (Université Paris VI)
Rationality questions for forms of moduli spaces, II

This is a continuation of the first talk. If time permits, I will also discuss some (positive) results, for rationality of F-forms of $\mathcal{M}_{g,n}$, in genus $g \geq 1$.

Attila Guld (Rényi Alfréd Matematikai Kutatóintózet)
Finite subgroups of the birational automorphism group are "almost nilpotent"

Investigating finite subgroups of the birational automorphism group of varieties was initiated by J.-P. Serre and V. L. Popov in the beginning of 2010's. It turned out to be a fruitful field of research.

The Jordan property lies in the center of attention. A group G is called Jordan, if there exists a constant J such that all finite subgroups of G has an Abelian subgroup of index at most J. In analogy we can introduce the notations of solvably and nilpotently Jordan properties by requiring a solvable or a nilpotent subgroup in the finite subgroups of G with small enough index.

Investigating surfaces Yu. G. Zarhin found that the birational automorphism group of a product of an elliptic curve and the projective line does not enjoy the Jordan property. (It turned out to be the only counterexample amongst surfaces.) On the contrary C. Shramov and Yu. Prokhorov showed that in many important cases the birational automorphism group is Jordan, moreover it is solvably Jordan for all varieties.

Therefore we know that the birational automorphism group is solvably Jordan, however it is not necessarily Jordan. Hence it is natural to ask what holds between the two properties. In my talk I will show that the birational automorphism group is nilpotently Jordan and give a bound for the nilpotency class in terms of the dimension of the variety.

Alexander Kuznetsov (Steklov Institute and HSE)
Rationality of prime Fano 3-folds over nonclosed fields

In the talk I will discuss rationality questions for forms of classical prime Fano 3-folds over nonclosed fields of characteristic 0.

Johannes Nicaise (Imperial College)
The non-archimedean SYZ fibration

This talk is based on joint work with Chenyang Xu and Tony Yue Yu. I will explain the construction of the non-archimedean Strominger–Yau–Zaslow fibration, whose existence was conjectured by Kontsevich and Soibelman in their non-archimedean approach to Mirror Symmetry. I will also explain why it is an affinoid torus fibration away from a codimension two subset of the base, as predicted by Kontsevich and Soibelman. The proof relies heavily on the Minimal Model Program in birational geometry.

John Ottem (University of Oslo)
Counterexamples to the integral Hodge conjecture

The Hodge conjecture predicts which rational cohomology classes on a smooth complex projective variety can be represented by linear combinations of complex subvarieties. The integral Hodge conjecture, the analogous conjecture for integral homology classes, is known to be false in general (the first counterexamples were given in dimension 7 by Atiyah and Hirzebruch). I'll survey some of the known results on this conjecture, and then present some new counterexamples. This is joint work with Olivier Benoist.

Stefan Schreieder (LMU München)
The rationality problem for hypersurfaces and quadric bundles

In this series of talks, I survey recent progress on the (stable) rationality problem for smooth projective hypersurfaces and quadric bundles. I explain in some detail Voisins degeneration method and its modifications due to Colliot-Thélène–Pirutka and myself. In order to apply this method, I recall some basic facts about unramified cohomology and explain the known strategies to construct unirational examples with nontrivial unramified cohomology. The latter originated in the work of Artin–Mumford (1972) and Colliot-Thélène–Ojanguren (1989) and has more recently been used in high degree by Asok (2013) and myself. Special emphasize will be given to an example of a quadric surface bundle over $\mathbb{P}^2$ with nontrivial unramified degree two cohomology due to Hassett–Pirutka–Tschinkel (2016), and its generalizations to higher dimensions and higher degree unramified cohomology, found by myself.

Andrey Trepalin (IITP and HSE)
Galois unirational surfaces

A surface X is called *unirational* if there exists a dominant rational map $\mathbb{P}^2 \dashrightarrow X$. For algebraically closed fields of characteristic zero any unirational surface is rational by Castelnuovo's rationality criterion. But if the ground field $\Bbbk$ is an arbitrary algebraically non-closed field then this does not hold. For example, any del Pezzo surface of degree 4, 3, or 2 (having a point defined over $\Bbbk$ in sufficiently general position) is unirational over $\Bbbk$, but some of these surfaces are not rational over $\Bbbk$.

A particular case of unirational surfaces is *Galois unirational* surfaces, that are surfaces birationally equivalent to quotients of rational surfaces. In the talk I will give a complete classification of Galois unirational surfaces over algebraically non-closed fields of characteristic zero, and discuss some related questions.

Vadim Vologodsky (HSE)
The Hochschild–Kostant–Rosenberg theorem fails in characteristic p (after Akhil Mathew)

Let X be a smooth algebraic variety over a field K, and let $\Delta\colon X \to X \times X$ be the diagonal embedding. Then the cohomology sheaves of the complex $\mathrm{L}\Delta^*\Delta_*\mathcal{O}_X$ are canonically identified with the sheaves of differential forms on X. In particular, there is a spectral sequence from the Hodge cohomology of X to the hypercohomology of the complex $\mathrm{L}\Delta^*\Delta_*\mathcal{O}_X$. If the characteristic of the base field K is 0 or larger then $\dim X$, the complex $\mathrm{L}\Delta^*\Delta_*\mathcal{O}_X$ is formal, i.e. quasi-isomorphic to the direct sum of its cohomology sheaves. It follows that in this case the above spectral sequence degenerates at the first page. It has been a longstanding question whether this degeneration holds in any characteristic. I will explain a recent result of Akhil Mathew showing that the analogous spectral sequence fails to degenerate for the classifying stack of the finite group scheme μ_p over $\mathbb{F}_p$. This easily yields an example of a smooth projective projective variety X such that the spectral sequence does not degenerate.

Egor Yasinsky (University of Basel)
Quotients of birational automorphism groups of rationally connected threefolds

Based on approach of Blanc–Lamy–Zimmermann, I will explain how to find a non-trivial surjective homomorphism from the birational automorphism group of a rationally connected variety to an abelian group. After explaining main ideas, I will focus on the case of del Pezzo fibrations, extending results of the mentioned authors. This is a work in progress with Jeremy Blanc.