

1 Show that if $P \neq NP$, then $P \neq PSPACE$.

2 Let coNPSpace be the class of languages $L \subseteq \{0, 1\}^*$ such that their complement $\bar{L} = \{0, 1\}^* \setminus L$ is in NPSpace . Show that $\text{coNPSpace} = \text{NPSpace}$.

3 Consider the problem HALF-CLIQUE .

Instance: An undirected graph G on an even number of vertices n .

Question: Is there a clique in G of size at least $n/2$?

Prove that HALF-CLIQUE is NP-complete.

4 For $x, y \in \{0, 1\}^n$ consider $f(x, y) = 1$ if x and y differ in all bits and $f(x, y) = 0$ otherwise. Show that $R(f) = O(1)$ (Alice and Bob have common random bits).

5 The matrix $A = \{a_{ij}\}_{i,j=1}^n \in \{0, 1\}^{n \times n}$ is *symmetrical* if for all i, j we have $a_{ij} = a_{ji}$. Show that there exists a tester to check that the matrix is symmetrical that uses $O(1/\epsilon)$ queries (two $n \times n$ matrices are ϵ -far if they differ in more than ϵn^2 entries).

6 Is it possible that for some $f: \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}$ we have $N^1(f) = \Theta(\log n)$, $N^0(f) = \Theta(\log n)$ and $D(f) = \Theta(n)$?

7 Suppose $A \in \text{BPP}$. By BPP^A we denote the class of languages that are decidable by randomized Turing machines with oracle access to A within polynomial time and with error probability at most $1/3$. Show that $\text{BPP}^A = \text{BPP}$.

8 Consider matrices $A \in \mathbb{Q}^{n \times n}$. Show that there exists a tester to check that every column in A is sorted (in the increasing order from top to bottom) that uses $O((\log n)/\epsilon)$ queries (two $n \times n$ matrices are ϵ -far if they differ in more than ϵn^2 entries).