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Letter to the Editor

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Page 30 of our article [1] (p. 685 of the English translation) contains several errors. Below a revised text is given for the part of the article on p. 685 of the English version beginning with the words "By remark (ii) above and Lemma 1," (third line from above) and ending with "for some absolute positive constant c. " (12th line from below):
By remark (ii) above and Lemma 1 we obtain

$$\left(\sum_{j=1}^{m}\left|\left(\frac{v_p}{\sqrt{N}}, \frac{W^{(j)}}{\sqrt{N}}\right) - \nu_j^p(\sigma)\right|^2\right)^{1/2} \geq \frac{1}{16}\left[\frac{1}{2^{16U(\sigma)}} - \frac{1}{m^{1/3}}\right],$$

provided that $U(\sigma) \leq (\log_2 m - 3)/32$, which allows us to use Lemma 1. Now if $U(\sigma) > (\log_2 m - 3)/32$, then the last inequality obviously holds since the right side becomes negative. In addition, by Bessel's inequality, we obtain

$$\left(\sum_{j=1}^{m}|(v_p, W^{(j)})|^2\right)^{1/2} \leq N.$$

Hence with probability $\geq 1 - 5/N^3$ we have

$$\begin{aligned}
&\left(\frac{2}{\varepsilon^2} + U(\sigma)\right)(20\delta N \log N)^{1/4} \\
&\quad\geq \left(\frac{2}{\varepsilon^2} + U(\sigma)\right)^{1/2} U(\sigma)^{1/2}(20\delta N \log N)^{1/4} \\
&\quad\geq \frac{(1-\varepsilon)(\delta N)^{1/2}}{16 \cdot 2^{16U(\sigma)}} - \frac{(1-\varepsilon)(\delta N)^{1/2}}{16 m^{1/3}} - 3(\varepsilon\delta N)^{1/2} - 4\varepsilon(20\delta N \log N)^{1/4}. \qquad (**)
\end{aligned}$$

To finish the proof of $(*)$ and hence of Theorem 1, it is enough to verify that the inequalities given above imply

$$U(\sigma) \geq c \log\left(\frac{\delta N}{\log N}\right) \qquad (***)$$

for some absolute positive constant c. If

$$U(\sigma) \geq \frac{1}{200} \log\left(\frac{\delta N}{\log N}\right),$$

then $(***)$ is obviously satisfied with $c = 1/200$. But if the last inequality is not satisfied, then

$$U(\sigma) \leq \frac{1}{54} \log_2 m$$

and

$$\frac{1}{2^{16U(\sigma)}} - \frac{1}{m^{1/3}} \geq \frac{2}{3}\frac{1}{2^{16U(\sigma)}}.$$

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In this case ($**$) yields

$$\left(\frac{2}{\varepsilon^2} + U(\sigma)\right)(20\delta N \log N)^{1/4} \geq \frac{(\delta N)^{1/2}}{30 \cdot 2^{16U(\sigma)}} - 4(\varepsilon\delta N)^{1/2}.$$

Let us show that the last inequality leads also to ($***$).

References

1. B. S. Kashin and L. A. Tsafriri, "Random sets of uniform convergence," *Mat. Zametki* [*Math. Notes*], **54**, No. 1, 17–33 [677–687] (1993).

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