Off-shell symmetry algebra of the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ superstring

DMITRI BYKOV

Trinity College Dublin, Ireland
Steklov Mathematical Institute Moscow, Russia

Mathematica Summer School
University of Porto, Porto, Portugal
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AHARONY, BERGMANN, JAFFERIS, MALDACENA 06"08'

$\mathcal{N}=6$ superconformal Chern-Simons-matter theories (N,k)

Discrete values of the coupling constant

$\Updownarrow$

$\mathrm{AdS}_4 \times \mathrm{S}^7/\mathbb{Z}_k$ solution of 11D supergravity

$$\mathrm{S}^7 \stackrel{\pi}{\to} \mathbb{C}\mathrm{P}^3; \quad \pi^{-1}(x) \sim S^1, \quad x \in \mathbb{C}\mathrm{P}^3$$

't Hooft limit $\quad N, k \to \infty, \quad \lambda \sim \dfrac{N}{k}$ fixed and real

IIA superstring theory on $\quad \mathrm{AdS}_4 \times \mathbb{C}\mathrm{P}^3$

String theory on $\mathrm{AdS}_4 \times \mathbb{C}\mathrm{P}^3$
as a sigma model on the coset

$$\frac{\mathrm{OSP}(2,2|6)}{\mathrm{SO}(3,1)\times \mathrm{U}(3)}$$

maximally
symmetric space

Maximal bosonic subgroup $\mathrm{USP}(2,2)\times \mathrm{SO}(6)$

24 real fermions

↓ κ -symmetry fixing

16 real fermions = # of physical bosonic d.o.f.

There exists a $\mathbb{Z}_4$-grading of the Lie algebra $\mathrm{osp}(2,2|6)$

Let $g \in OSP(2,2|6)$

Define a left-invariant $\mathrm{osp}(2,2|6)$ -valued 1-form $\mathrm{A} \equiv g^{-1}dg$

Its decomposition under the grading gives

$\mathrm{A}^{(2)}$ - vielbein (zehnbein)

$\mathrm{A}^{(0)}$ - spin connection

$\mathrm{A}^{(1)},\ \mathrm{A}^{(3)}$ - fermionic components

The Lagrangian

$$\mathcal{L} = \gamma^{\alpha\beta}\mathrm{str}\big(A_\alpha^{(2)}A_\beta^{(2)}\big) + \kappa\epsilon^{\alpha\beta}\mathrm{str}\big(A_\alpha^{(1)}A_\beta^{(3)}\big)$$

$\kappa = \pm 1$ by κ-symmetry

Coset parameterization

$$g = g_{\rm A} g_\chi g_{\rm B}$$

fermionic elements

Global symmetry group acts from the left $g \to g_0 g, \; g_0 \in {\rm OSP}(2,2|6)$

whereas local κ-symmetry acts from the right $g \to g e^\epsilon, \; \epsilon$ constrained

ARUTYUNOV, FROLOV 06″08′

$g_{\rm A}$ represents some submanifold in the coset, on which group ${\rm H} \in {\rm OSP}(2,2|6)$ multiplication can be defined

⇓

Fermions χ are neutral under ${\rm H}$

A closer look at $\mathbb{CP}^3$

Orthogonal complex structures in $\mathbb{R}^6$

Pick the simplest one $K_6 = I_3 \otimes i\sigma_2$

$\omega K_6 \omega^{-1}$ again a complex structure

$\omega = 1 + \epsilon + \ldots$

$$\longrightarrow \quad \{K_6, [\epsilon, K_6]\} = 0$$

$$f(a) = [a, K_6] \qquad g(b) \equiv \{K_6, b\}$$

$$\boxed{0 \longrightarrow u(3) \xrightarrow{i} o(6) \xrightarrow{f} o(6) \xrightarrow{g} \mathbb{R}^N}$$

Matrices describing the complex projective space

```
EE[i_, j_] := KroneckerProduct[e_i, e_j];
T1 := EE[1, 3] - EE[3, 1] - EE[2, 4] + EE[4, 2];
T2 := EE[1, 4] - EE[4, 1] - EE[3, 2] + EE[2, 3];
T3 := EE[1, 5] - EE[5, 1] - EE[2, 6] + EE[6, 2];
T4 := EE[1, 6] - EE[6, 1] - EE[5, 2] + EE[2, 5];
T5 := EE[3, 5] - EE[5, 3] - EE[4, 6] + EE[6, 4];
T6 := EE[3, 6] - EE[6, 3] - EE[5, 4] + EE[4, 5];
```

Light-cone directions

$$\Xi_+ := \text{ArrayFlatten}\left[\left(\begin{array}{c|c} \Gamma_0 & 0 \\ \hline 0 & -I\,T_6 \end{array}\right)\right];\quad \Xi_- := \text{ArrayFlatten}\left[\left(\begin{array}{c|c} -\Gamma_0 & 0 \\ \hline 0 & -I\,T_6 \end{array}\right)\right];$$

Light-cone prefactor

$$\Lambda := \text{MatrixExp}\left[\frac{I}{2}\left(\text{xp}\,\Xi_+ + \frac{1}{2}\,\text{xm}\,\Xi_-\right)\right]$$

Matrix exponentiation

```
MatrixForm[FullSimplify[MatrixExp[dz T5]]]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \text{Cos}[dz] & 0 & \text{Sin}[dz] & 0 \\ 0 & 0 & 0 & \text{Cos}[dz] & 0 & -\text{Sin}[dz] \\ 0 & 0 & -\text{Sin}[dz] & 0 & \text{Cos}[dz] & 0 \\ 0 & 0 & 0 & \text{Sin}[dz] & 0 & \text{Cos}[dz] \end{pmatrix}$$

Light-cone gauge

Light-cone coordinates $x_+ = \frac{1}{2}(\varphi + t), \;\; x_- = \varphi - t$

The coset $g = g_O g_\chi g_B$ $\qquad g_O = \exp\left(\frac{i}{2}t\Gamma_0 + \frac{\varphi}{2}T_6\right)$

$$g_{\mathrm{AdS}} = \frac{1}{\sqrt{1+\frac{z^2}{4}}}\left(1 + \frac{i}{2}\sum_{i=1}^{3} z_i \Gamma_i\right)$$

4 | 6 x 4 | 6 dimensional matrices

$$g_{\mathbb{CP}} = I + \frac{1}{\sqrt{1+|w|^2}}(W + \bar{W}) + \frac{\sqrt{1+|w|^2}-1}{|\omega|^2\sqrt{1+|w|^2}}(W\bar{W} + \bar{W}W)$$

The gauge $\qquad x_+ = \tau \qquad p_+ = 1$

Residual AdS matrix

$$\text{Gads} := \frac{1}{\sqrt{1 - \text{eps}^2 \frac{\sum_{i=1}^{3} z_i^2}{4}}} \left(\text{IdentityMatrix}[4] + \text{eps}\, \frac{I}{2} \sum_{i=1}^{3} z_i\, \Gamma_i \right);$$

$$\text{Tau1} := \frac{T_1 - I\, T_2}{2};\ \text{Tau2} := \frac{T_3 - I\, T_4}{2};\ \overline{\text{Tau1}} := \text{Conjugate}[\text{Tau1}];\ \overline{\text{Tau2}} := \text{Conjugate}[\text{Tau2}];$$

$$W := \frac{1}{2} (w_1\, \text{Tau1} + w_2\, \text{Tau2});\ \bar{W} := \frac{1}{2} \left(\bar{w}_1\, \overline{\text{Tau1}} + \bar{w}_2\, \overline{\text{Tau2}}\right);$$

$$\text{Wmod} := \frac{1}{4} (w_1\, \bar{w}_1 + w_2\, \bar{w}_2);$$

CP3-related parameterizations and conversion to TeX

```
MatrixForm[W] // TeXForm

\left(
\begin{array}{llllll}
 0 & 0 & \frac{w_1}{4} & -\frac{i w_1}{4} & \frac{w_2}{4} & -\frac{i w_2}{4} \\
 0 & 0 & -\frac{i w_1}{4} & -\frac{w_1}{4} & -\frac{i w_2}{4} & -\frac{w_2}{4} \\
 -\frac{w_1}{4} & \frac{i w_1}{4} & 0 & 0 & 0 & 0 \\
 \frac{i w_1}{4} & \frac{w_1}{4} & 0 & 0 & 0 & 0 \\
 -\frac{w_2}{4} & \frac{i w_2}{4} & 0 & 0 & 0 & 0 \\
 \frac{i w_2}{4} & \frac{w_2}{4} & 0 & 0 & 0 & 0
\end{array}
\right)
```

Residual CP3 matrix

$$\text{Gcp} := \text{IdentityMatrix}[6] + \frac{\text{eps}}{\sqrt{1 + \text{eps}^2\, \text{Wmod}}} (W + \bar{W}) + \frac{\sqrt{1 + \text{eps}^2\, \text{Wmod}} - 1}{\text{Wmod}\, \sqrt{1 + \text{eps}^2\, \text{Wmod}}} (W.\bar{W} + \bar{W}.W)$$

The kappa-symmetry gauge

DE AZCARRAGA, LUKIERSKI 1982
SIEGEL, 1983
GREEN, SCHWARZ 1984

Infinitesimal kappa-transformation

$$\chi = \begin{bmatrix} 0 & \theta \\ \eta & 0 \end{bmatrix} \qquad \delta\theta = \begin{bmatrix} 0 & 0 & \epsilon_1 & \epsilon_2 & -i\epsilon_2 & -i\epsilon_1 \\ 0 & 0 & \epsilon_3 & \epsilon_4 & -i\epsilon_4 & -i\epsilon_3 \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \end{bmatrix}$$

$$f_1(\vartheta) \equiv [\vartheta, \Sigma_+]$$

The variation is in the kernel

Gauge equivalence classes labeled by

$$W_F/\mathrm{Ker} f_1 \sim \mathrm{Im}\, f_1$$

The gauge $\chi = [\Sigma_+, \xi]$

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Fermions at the start

```
JJ[i_, j_] := Outer[Times, UnitVector[4, i], UnitVector[6, j]];
χ := Sum[n_{i,j} JJ[i, j], {i, 1, 4}, {j, 1, 6}];
```

$$CC_4 := \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix};$$

```
η := -Transpose[χ].CC4;
```

$$\text{ferm} := \text{ArrayFlatten}\left[\begin{pmatrix} 0\,\text{IdentityMatrix}[4] & \chi \\ \eta & 0\,\text{IdentityMatrix}[6] \end{pmatrix}\right];$$

```
MatrixForm[ferm]
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & n_{1,1} & n_{1,2} & n_{1,3} & n_{1,4} & n_{1,5} & n_{1,6} \\
0 & 0 & 0 & 0 & n_{2,1} & n_{2,2} & n_{2,3} & n_{2,4} & n_{2,5} & n_{2,6} \\
0 & 0 & 0 & 0 & n_{3,1} & n_{3,2} & n_{3,3} & n_{3,4} & n_{3,5} & n_{3,6} \\
0 & 0 & 0 & 0 & n_{4,1} & n_{4,2} & n_{4,3} & n_{4,4} & n_{4,5} & n_{4,6} \\
n_{4,1} & -n_{3,1} & n_{2,1} & -n_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
n_{4,2} & -n_{3,2} & n_{2,2} & -n_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 \\
n_{4,3} & -n_{3,3} & n_{2,3} & -n_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\
n_{4,4} & -n_{3,4} & n_{2,4} & -n_{1,4} & 0 & 0 & 0 & 0 & 0 & 0 \\
n_{4,5} & -n_{3,5} & n_{2,5} & -n_{1,5} & 0 & 0 & 0 & 0 & 0 & 0 \\
n_{4,6} & -n_{3,6} & n_{2,6} & -n_{1,6} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Fermionic multiplets

```
fermFullMat :=
```

$$\begin{pmatrix}
\kappa_{1,1} - \kappa_{2,-1} & -i\,(\kappa_{1,1} + \kappa_{2,-1}) & \frac{1}{2}(s_{1,1} - t_{1,2}) & -\frac{1}{2} i\,(s_{1,1} + t_{1,2}) & \frac{1}{2}(s_{1,2} + t_{1,1}) & -\frac{1}{2} i\,(s_{1,2} - t_{1,1}) \\
\kappa_{1,-1} + \kappa_{2,1} & i\,(\kappa_{1,-1} - \kappa_{2,1}) & \frac{1}{2}(s_{2,1} - t_{2,2}) & -\frac{1}{2} i\,(s_{2,1} + t_{2,2}) & \frac{1}{2}(s_{2,2} + t_{2,1}) & -\frac{1}{2} i\,(s_{2,2} - t_{2,1}) \\
-\bar{\kappa}_{1,-1} - \bar{\kappa}_{2,1} & i\,(\bar{\kappa}_{1,-1} - \bar{\kappa}_{2,1}) & \frac{1}{2}(-\bar{s}_{2,1} + \bar{t}_{2,2}) & -\frac{1}{2} i\,(\bar{s}_{2,1} + \bar{t}_{2,2}) & \frac{1}{2}(-\bar{s}_{2,2} - \bar{t}_{2,1}) & -\frac{1}{2} i\,(\bar{s}_{2,2} - \bar{t}_{2,1}) \\
\bar{\kappa}_{1,1} - \bar{\kappa}_{2,-1} & i\,(\bar{\kappa}_{1,1} + \bar{\kappa}_{2,-1}) & \frac{1}{2}(\bar{s}_{1,1} - \bar{t}_{1,2}) & \frac{1}{2} i\,(\bar{s}_{1,1} + \bar{t}_{1,2}) & \frac{1}{2}(\bar{s}_{1,2} + \bar{t}_{1,1}) & \frac{1}{2} i\,(\bar{s}_{1,2} - \bar{t}_{1,1})
\end{pmatrix}$$

Supercharges

The supercurrent

$$J^\alpha = g\left(\gamma^{\alpha\beta}A^{(2)}_\beta + \frac{\kappa}{2}\epsilon^{\alpha\beta}(A^{(3)}_\beta - A^{(1)}_\beta)\right)g^{-1}$$

DE WIT, FREEDMAN 1975

Once the kappa gauge has been imposed,
the action of supersymmetry transformations is modified:

$$g \to e^{\epsilon} g e^{\tilde{\epsilon}}$$

The supercharges

$$Q^a_\alpha = \frac{i}{4}\int d\sigma\, e^{-i\frac{x_-}{2}}\Big(2p_y\,\chi^a_\alpha + 2\epsilon^{ab}(Z^*)_b^{\ c}(\epsilon_{\alpha\beta}\bar{\chi}^\beta_c + i\epsilon_{cd}\chi'^d_\alpha) -$$
$$- 2i\epsilon^{ab}(P^*_z)_b^{\ c}\epsilon_{\alpha\beta}\bar{\chi}^\beta_c - i\epsilon_{\alpha\beta}\bar{w}^\beta(\kappa^{a,+1} - 2i(\bar{\kappa}')^{a,-1}) - i\epsilon^{ab}w_\alpha(\kappa_b^{-1} - 2i(\bar{\kappa}')_b^{+1}) +$$
$$+ 2\epsilon^{ab}P_{w,\alpha}\kappa_b^{-1} + 2\epsilon_{\alpha\beta}\bar{P}^\beta_w\kappa^{a,+1} - 2i\,y\,(\chi^a_\alpha + i\epsilon^{ab}\epsilon_{\alpha\beta}(\bar{\chi}')^\beta_b)\Big)$$

x'_- determined from the Virasoro conditions

The full coset matrix

```
GCoset := Λ.FermCoset.YEmbed.GadsEmbed.GcpEmbed;
```

The sigma-derivative of this matrix

$$dG := D[GCoset, y[1]]\, y'[1] + D[GCoset, xm]\, pxm + \sum_{i=1}^{3} D[GCoset, z_i]\, z'_i + \sum_{j=1}^{2} D[GCoset, \varkappa_{j,-1}]\, \varkappa'_{j,-1} + \sum_{j=1}^{2} D[GCoset, \varkappa_{j,1}]\, \varkappa'_{j,1} + \sum_{j=1}^{2} D[GCoset, \bar{\varkappa}_{j,-1}]\, \bar{\varkappa}'_{j,-1} + \sum_{j=1}^{2} D[GCoset, \bar{\varkappa}_{j,1}]\, \bar{\varkappa}'_{j,1} + \sum_{i=1}^{2}\sum_{j=1}^{2} D[GCoset, s_{j,i}]\, s'_{j,i} + \sum_{i=1}^{2}\sum_{j=1}^{2} D[GCoset, \bar{s}_{j,i}]\, \bar{s}'_{j,i} + \sum_{i=1}^{2} D[GCoset, w_i]\, w'_i + \sum_{i=1}^{2} D[GCoset, \bar{w}_i]\, \bar{w}'_i$$

The left-invariant current

$$C_1 := ArrayFlatten\left[\begin{pmatrix} CC_4 & 0 \\ 0 & IdentityMatrix[6] \end{pmatrix}\right];$$

```
ST[qq_] := Table[If[(ii < 5 && jj < 5) || (ii > 4 && jj > 4), 1, 0] qq[[jj, ii]] -
    If[ii < 5 && jj > 4, 1, 0] qq[[jj, ii]] + If[ii > 4 && jj < 5, 1, 0] qq[[jj, ii]], {ii, 10}, {jj, 10}];
Inv[gg_] := Inverse[C1].ST[gg].C1;

Current := Simplify[Series[-Inv[GCoset].dG, {eps, 0, 2}]];
```

Bosonic charges

```
Do[
  BLCharge[ss, tt] =
   I/2 Simplify[Normal[Series[STra[L[ss, tt].GCoset.Noether.Inv[GCoset]] /. pminsub /. pplussub,
       {eps, 0, 2}]]], {ss, 1, 2}, {tt, 1, 2}];
```

A typical fermionic charge

```
FCharge[1, 1] =
  I/2 Normal[Series[STra[M[1, 1].GCoset.Noether.Inv[GCoset]] /. pminsub /. pplussub /. {pxm → 0},
     {eps, 0, 2}]];
```

The result

```
FullSimplify[FCharge[1, 1]]
```

$$\frac{1}{4} e^{\frac{i\,xm}{2}} eps^2 \Big(w_2 \big(\bar{\kappa}_{1,1} + 2 i (\kappa')_{1,-1}\big) + \bar{w}_1 \big(\bar{\kappa}_{2,-1} + 2 i (\kappa')_{2,1}\big) + 2 i z_3 \big(s_{2,2} + i (\bar{s}')_{1,1}\big) + z_1 \big(-2 i s_{1,2} - 2 (\bar{s}')_{2,1}\big) + 2 z_2 \big(s_{1,2} - i (\bar{s}')_{2,1}\big) - 2 i \big(P_y \bar{s}_{1,1} + \big(i \bar{s}_{1,1} + (s')_{2,2}\big) y[1]\big) - 2 s_{2,2} P_z[3] - 2 i \big(\bar{\kappa}_{1,1} P_w[2] + i s_{1,2} (P_z[1] + i P_z[2]) + \bar{\kappa}_{2,-1} \bar{P}_w[1]\big)\Big)$$

The central extension

The centrally-extended $\mathrm{su}(2|2) \oplus \mathrm{u}(1)$:

$$
\begin{aligned}
[\mathrm{R}_\alpha^\beta, \mathrm{R}_\gamma^\delta] &= \delta_\alpha^\delta \mathrm{R}_\gamma^\beta - \delta_\gamma^\beta \mathrm{R}_\alpha^\delta \\
[\mathrm{L}_a^b, \mathrm{L}_c^d] &= \delta_a^d \mathrm{L}_c^b - \delta_c^b \mathrm{L}_a^d \\
\{\mathcal{Q}_\alpha^a, \bar{\mathcal{Q}}_b^\beta\} &= \delta_\alpha^\beta \mathrm{L}_b^a - \delta_b^a \mathrm{R}_\alpha^\beta + \frac{1}{2}\delta_b^a \delta_\alpha^\beta \mathrm{H} \\
\{\mathcal{Q}_\alpha^a, \mathcal{Q}_\beta^b\} &= \epsilon_{\alpha\beta}\epsilon^{ab}\mathrm{P}_1 \\
\{\bar{\mathcal{Q}}_a^\alpha, \bar{\mathcal{Q}}_b^\beta\} &= \epsilon_{ab}\epsilon^{\alpha\beta}\mathrm{P}_2 .
\end{aligned}
$$

$$P_1 = -\frac{i}{2}\int d\sigma\, e^{-ix_-}\, x'_- = \frac{1}{2}e^{-ix_-(-\infty)}(e^{-ip}-1) = \frac{\xi}{2}(e^{-ip}-1)$$

$$P_2 = \bar{P}_1$$

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The dispersion relation follows $\epsilon(p) = \sqrt{1 + 16 h(\lambda)^2 \sin^2(\frac{p}{2})}$

```
PRIME[A_] := Simplify[Sum[D[A, z_i] z'_i, {i, 1, 3}]] + Simplify[Sum[D[A, P_z[i]] P_z'[i], {i, 1, 3}]] +
   Simplify[Sum[D[A, w_i] w'_i, {i, 1, 2}]] + Simplify[Sum[D[A, w̄_i] w̄'_i, {i, 1, 2}]] +
   D[A, P_w[1]] P_w'[1] + D[A, P_w[2]] P_w'[2] + D[A, P̄_w[1]] P̄_w'[1] + D[A, P̄_w[2]] P̄_w'[1] + D[A, P_y] P_y' +
   D[A, y[1]] y'[1] + D[A, xm] pxm;
PoissonBracket[A_, B_] :=
 Simplify[
  Sum[D[A, s_{i,j}] D[B, s̄_{i,j}] + D[A, s̄_{i,j}] D[B, s_{i,j}] - D[A, s_{i,j}] PRIME[D[B, s̄'_{i,j}]] -
     PRIME[D[A, s̄'_{i,j}]] D[B, s_{i,j}] - D[A, s̄_{i,j}] PRIME[D[B, s'_{i,j}]] - PRIME[D[A, s'_{i,j}]] D[B, s̄_{i,j}] +
     PRIME[D[A, s'_{i,j}]] PRIME[D[B, s̄'_{i,j}]] + PRIME[D[A, s̄'_{i,j}]] PRIME[D[B, s'_{i,j}]], {i, 1, 2}, {j, 1, 2}] +
   Sum[(D[A, κ_{i,-1+2 (j-1)}] D[B, κ̄_{i,-1+2 (j-1)}] + D[A, κ̄_{i,-1+2 (j-1)}] D[B, κ_{i,-1+2 (j-1)}] -
      D[A, κ_{i,-1+2 (j-1)}] PRIME[D[B, κ̄'_{i,-1+2 (j-1)}]] - PRIME[D[A, κ̄'_{i,-1+2 (j-1)}]] D[B, κ_{i,-1+2 (j-1)}] -
      D[A, κ̄_{i,-1+2 (j-1)}] PRIME[D[B, κ'_{i,-1+2 (j-1)}]] - PRIME[D[A, κ'_{i,-1+2 (j-1)}]] D[B, κ̄_{i,-1+2 (j-1)}] +
      PRIME[D[A, κ'_{i,-1+2 (j-1)}]] PRIME[D[B, κ̄'_{i,-1+2 (j-1)}]] +
      PRIME[D[A, κ̄'_{i,-1+2 (j-1)}]] PRIME[D[B, κ'_{i,-1+2 (j-1)}]]), {i, 1, 2}, {j, 1, 2}]]
```

Calculation and printing of Poisson brackets of the supercharges

```
Do[Print["{Q[" , ii, ",", jj, "],Q̄[" , s, ",", t, "]} =",
  Simplify[PoissonBracket[1/eps FCharge[ii, jj], 1/eps FconCharge[s, t]] -
    KroneckerDelta[ii, s] BRCharge[t, jj] + KroneckerDelta[t, jj] BLCharge[s, ii] -
    1/2 KroneckerDelta[ii, s] KroneckerDelta[t, jj] Hamiltonian] /. {pxm → 0, λ → 1}, ";"],
 {ii, 1, 2}, {jj, 1, 2}, {s, 1, 2}, {t, 1, 2}]
```

```
Do[Print["{Q[" , ii, ",", jj, "],Q[" , s, ",", t, "]} =",
  Simplify[PoissonBracket[1/eps FCharge[ii, jj], 1/eps FCharge[s, t]] -
    1/2 e^(I xm) pxmsubB eps[ii, s] eps[jj, t] /. {pxm → 0}], ";"], {ii, 1, 2}, {jj, 1, 2},
 {s, 1, 2}, {t, 1, 2}]
```

The result

```
{Q[1,1],Q[1,1]} =0;
{Q[1,1],Q[1,2]} =0;
{Q[1,1],Q[2,1]} =0;
{Q[1,1],Q[2,2]} =
```

$$\frac{1}{8} i e^{i\,xm} eps^2 (\bar{w}_1 (w')_1 + \bar{w}_2 (w')_2 - 4 z_1 (z')_1 - 4 z_2 (z')_2 - 4 z_3 (z')_3 + w_1 (\bar{w}')_1 + w_2 (\bar{w}')_2 + 4 y[1] y'[1]);$$

Conclusions and outlook

- Kappa-symmetry gauge which does not break bosonic symmetries of the light-cone gauge

- Central extension as a function of world-sheet momentum

- Loop corrections to the central extension?

- Calculation of the S-matrix?