Off-shell symmetry algebra of $\mathrm{AdS}_4 \times \mathbb{CP}^3$ superstring

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AHARONY, BERGMANN, JAFFERIS, MALDACENA 06"08'

$\mathcal{N}=6$ superconformal Chern-Simons-matter theories (N,k)

$\Updownarrow$

Discrete values of the coupling constant

$\mathrm{AdS}_4 \times \mathrm{S}^7/\mathbb{Z}_k$ solution of 11D supergravity

$$\mathrm{S}^7 \xrightarrow{\pi} \mathbb{C}\mathrm{P}^3; \quad \pi^{-1}(x) \sim S^1, \quad x \in \mathbb{C}\mathrm{P}^3$$

't Hooft limit $N, k \to \infty, \quad \lambda \sim \frac{N}{k}$ fixed and real

IIA superstring theory on $\mathrm{AdS}_4 \times \mathbb{C}\mathrm{P}^3$

String theory on $AdS_4 \times \mathbb{C}P^3$ ← maximally symmetric space

as a sigma model on the coset

$$\frac{OSP(2,2|6)}{SO(3,1) \times U(3)}$$

Maximal bosonic subgroup $USP(2,2) \times SO(6)$

24 real supercharges

↓ κ -symmetry fixing

16 real supercharges = # of physical bosonic d.o.f.

There exists a $\mathbb{Z}_4$ -grading of the Lie algebra $\mathrm{osp}(2,2|6)$

Let $g \in OSP(2,2|6)$

Define a left-invariant $\mathrm{osp}(2,2|6)$ -valued 1-form $\mathrm{A} \equiv g^{-1}dg$

Its decomposition under the grading gives

$\mathrm{A}^{(2)}$ - vielbein (zehnbein)

$\mathrm{A}^{(0)}$ - spin connection

$\mathrm{A}^{(1)},\ \mathrm{A}^{(3)}$ - fermionic components

The Lagrangian

$$\mathcal{L} = \gamma^{\alpha\beta}\mathrm{str}\big(A^{(2)}_\alpha A^{(2)}_\beta\big) + \kappa\epsilon^{\alpha\beta}\mathrm{str}\big(A^{(1)}_\alpha A^{(3)}_\beta\big)$$

$\kappa = \pm 1$ by κ -symmetry

Coset parameterization

$$g = g_{\rm A} g_\chi g_{\rm B}$$

↑ fermionic elements

Global symmetry group acts from the left $g \to g_0 g, \;\; g_0 \in {\rm OSP}(2,2|6)$

whereas local κ -symmetry acts from the right $g \to g e^{\epsilon}, \;\; \epsilon$ constrained

ARUTYUNOV, FROLOV 06"08'

$g_{\rm A}$ represents some submanifold in the coset, on which group ${\rm H} \in {\rm OSP}(2,2|6)$ multiplication can be defined

⇓

Fermions χ are neutral under ${\rm H}$

A closer look at $\mathbb{CP}^3$

Orthogonal complex structures in $\mathbb{R}^6$

Pick the simplest one $K_6 = I_3 \otimes i\sigma_2$

$\omega K_6 \omega^{-1}$ again a complex structure

$\omega = 1 + \epsilon + \ldots$

$$\{K_6, [\epsilon, K_6]\} = 0$$

$$f(a) = [a, K_6] \qquad g(b) \equiv \{K_6, b\}$$

$$\boxed{0 \longrightarrow u(3) \xrightarrow{i} o(6) \xrightarrow{f} o(6) \xrightarrow{g} \mathbb{R}^N}$$

Light-cone gauge

Light-cone coordinates $x_+ = \frac{1}{2}(\varphi + t), \;\; x_- = \varphi - t$

The coset $g = g_O g_\chi g_B$ $\qquad g_O = \exp\left(\frac{i}{2} t\Gamma_0 + \frac{\varphi}{2} T_6\right)$

$$g_{\mathrm{AdS}} = \frac{1}{\sqrt{1+\frac{z^2}{4}}}\left(1 + \frac{i}{2}\sum_{i=1}^{3} z_i \Gamma_i\right)$$

4 | 6 x 4 | 6 dimensional matrices

$$g_{\mathbb{CP}} = I + \frac{1}{\sqrt{1+|w|^2}}(W + \bar{W}) + \frac{\sqrt{1+|w|^2}-1}{|\omega|^2\sqrt{1+|w|^2}}(W\bar{W} + \bar{W}W)$$

The gauge $\qquad x_+ = \tau \qquad p_+ = 1$

The kappa-symmetry gauge

DE AZCARRAGA, LUKIERSKI 1982
SIEGEL, 1983
GREEN, SCHWARZ 1984

Infinitesimal kappa-transformation

$$\chi = \begin{bmatrix} 0 & \theta \\ \eta & 0 \end{bmatrix} \qquad \delta\theta = \begin{bmatrix} 0 & 0 & \epsilon_1 & \epsilon_2 & -i\epsilon_2 & -i\epsilon_1 \\ 0 & 0 & \epsilon_3 & \epsilon_4 & -i\epsilon_4 & -i\epsilon_3 \\ 0 & 0 & \star & \star & \star & \star \\ 0 & 0 & \star & \star & \star & \star \end{bmatrix}$$

$$f_1(\vartheta) \equiv [\vartheta, \Sigma_+]$$

The variation is in the kernel

Gauge equivalence classes labeled by

$$W_F/\mathrm{Ker} f_1 \sim \mathrm{Im}\, f_1$$

The gauge $\chi = [\Sigma_+, \xi]$

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Supercharges

The supercurrent

$$J^\alpha = g\left(\gamma^{\alpha\beta}A_\beta^{(2)} + \frac{\kappa}{2}\epsilon^{\alpha\beta}\left(A_\beta^{(3)} - A_\beta^{(1)}\right)\right)g^{-1}$$

DE WIT, FREEDMAN 1975

Once the kappa gauge has been imposed,
the action of supersymmetry transformations is modified:

$$g \to e^{\epsilon} g e^{\widetilde{\epsilon}}$$

The supercharges

$$\begin{aligned} Q_\alpha^a = \frac{i}{4}\int d\sigma\, e^{-i\,\frac{x_-}{2}}\, &\Big(2p_y\,\chi_\alpha^a + 2\epsilon^{ab}(Z^*)_b^{\ c}(\epsilon_{\alpha\beta}\bar{\chi}_c^\beta + i\epsilon_{cd}\chi_\alpha'^d) - \\ &- 2i\epsilon^{ab}(P_z^*)_b^{\ c}\epsilon_{\alpha\beta}\bar{\chi}_c^\beta - i\epsilon_{\alpha\beta}\bar{w}^\beta(\kappa^{a,+1} - 2i(\bar{\kappa}')^{a,-1}) - i\epsilon^{ab}w_\alpha(\kappa_b^{-1} - 2i(\bar{\kappa}')_b^{+1}) + \\ &+ 2\epsilon^{ab}P_{w,\alpha}\kappa_b^{-1} + 2\epsilon_{\alpha\beta}\bar{P}_w^\beta\kappa^{a,+1} - 2i\,y\,(\chi_\alpha^a + i\epsilon^{ab}\epsilon_{\alpha\beta}(\bar{\chi}')_b^\beta)\Big) \end{aligned}$$

x'_- determined from the Virasoro conditions

The central extension

The centrally-extended $\mathrm{su}(2|2) \oplus \mathrm{u}(1)$

:

$$[\mathrm{R}_\alpha^\beta, \mathrm{R}_\gamma^\delta] = \delta_\alpha^\delta \mathrm{R}_\gamma^\beta - \delta_\gamma^\beta \mathrm{R}_\alpha^\delta$$

$$[\mathrm{L}_a^b, \mathrm{L}_c^d] = \delta_a^d \mathrm{L}_c^b - \delta_c^b \mathrm{L}_a^d$$

$$\{\mathcal{Q}_\alpha^a, \bar{\mathcal{Q}}_b^\beta\} = \delta_\alpha^\beta \mathrm{L}_b^a - \delta_b^a \mathrm{R}_\alpha^\beta + \frac{1}{2}\delta_b^a \delta_\alpha^\beta \mathrm{H}$$

$$\{\mathcal{Q}_\alpha^a, \mathcal{Q}_\beta^b\} = \epsilon_{\alpha\beta}\epsilon^{ab}\mathrm{P}_1$$

$$\{\bar{\mathcal{Q}}_a^\alpha, \bar{\mathcal{Q}}_b^\beta\} = \epsilon_{ab}\epsilon^{\alpha\beta}\mathrm{P}_2.$$

$$P_1 = -\frac{i}{2}\int d\sigma\, e^{-ix_-}\, x'_- = \frac{1}{2}e^{-ix_-(-\infty)}(e^{-ip} - 1) = \frac{\xi}{2}(e^{-ip} - 1)$$

$$P_2 = \bar{P}_1$$

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The dispersion relation follows $\epsilon(p) = \sqrt{1 + 16h(\lambda)^2 \sin^2(\frac{p}{2})}$

Conclusions and outlook

- Kappa-symmetry gauge which does not break bosonic symmetries of the light-cone gauge

- Central extension as a function of world-sheet momentum

- Loop corrections to the central extension?

- Calculation of the S-matrix?