

Off-shell symmetry algebra
of $AdS_4 \times CP^3$
superstring

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16 Irish Quantum Field Theory meeting
Trinity College, Dublin
15 May 2009

$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories (N, k)
 \Updownarrow
 $\text{AdS}_4 \times S^7 / \mathbb{Z}_k$ solution of 11D supergravity
 $S^7 \xrightarrow{\pi} \mathbb{CP}^3; \quad \pi^{-1}(x) \sim S^1, \quad x \in \mathbb{CP}^3$
Discrete values of the coupling constant

't Hooft limit $N, k \rightarrow \infty, \quad \lambda \sim \frac{N}{k}$ fixed and real

IIA superstring theory on $\text{AdS}_4 \times \mathbb{CP}^3$

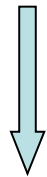
String theory on $\text{AdS}_4 \times \mathbb{CP}^3$
 as a sigma model on the coset

$$\frac{\text{OSP}(2, 2|6)}{\text{SO}(3, 1) \times \text{U}(3)}$$

maximally
 symmetric space

Maximal bosonic subgroup $\text{USP}(2, 2) \times \text{SO}(6)$

24 real supercharges



κ -symmetry fixing

16 real supercharges = # of physical bosonic d.o.f.

There exists a \mathbb{Z}_4 -grading of the Lie algebra $\mathfrak{osp}(2, 2|6)$

Let $g \in OSP(2, 2|6)$

Define a left-invariant $\mathfrak{osp}(2, 2|6)$ -valued 1-form $A \equiv g^{-1}dg$

Its decomposition under the grading gives

$A^{(2)}$ - vielbein (zehnbein)

$A^{(0)}$ - spin connection

$A^{(1)}, A^{(3)}$ - fermionic components

The Lagrangian

$$\mathcal{L} = \gamma^{\alpha\beta} \text{str}(A_{\alpha}^{(2)} A_{\beta}^{(2)}) + \kappa \epsilon^{\alpha\beta} \text{str}(A_{\alpha}^{(1)} A_{\beta}^{(3)})$$

$$\kappa = \pm 1 \quad \text{by } \kappa \text{ -symmetry}$$

Coset parameterization

$$g = g_A g_\chi g_B$$



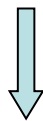
fermionic elements

Global symmetry group acts from the left $g \rightarrow g_0 g$, $g_0 \in \text{OSP}(2, 2|6)$

whereas local \mathfrak{kl} -symmetry acts from the right $g \rightarrow g e^\epsilon$, ϵ constrained

ARUTYUNOV, FROLOV 06''08'

g_A represents some submanifold in the coset,
on which group $H \in \text{OSP}(2, 2|6)$ multiplication can be defined



Fermions χ are neutral under H


A closer look at $\mathbb{C}P^3$

Orthogonal complex structures in \mathbb{R}^6

Pick the simplest one $K_6 = I_3 \otimes i\sigma_2$

$\omega K_6 \omega^{-1}$ again a complex structure

$$\omega = 1 + \epsilon + \dots$$


$$\{K_6, [\epsilon, K_6]\} = 0$$

$$f(a) = [a, K_6] \quad g(b) \equiv \{K_6, b\}$$

$$0 \longrightarrow u(3) \xrightarrow{i} o(6) \xrightarrow{f} o(6) \xrightarrow{g} \mathbb{R}^N$$

Light-cone gauge

Light-cone coordinates $x_+ = \frac{1}{2}(\varphi + t), \quad x_- = \varphi - t$

The coset $g = g_O g_X g_B$ $g_O = \exp\left(\frac{i}{2}t\Gamma_0 + \frac{\varphi}{2}T_6\right)$

$$g_{\text{AdS}} = \frac{1}{\sqrt{1 + \frac{z^2}{4}}} \left(1 + \frac{i}{2} \sum_{i=1}^3 z_i \Gamma_i \right)$$

4 | 6 x 4 | 6
dimensional
matrices

$$g_{\text{CP}} = I + \frac{1}{\sqrt{1 + |w|^2}} (W + \bar{W}) + \frac{\sqrt{1 + |w|^2} - 1}{|w|^2 \sqrt{1 + |w|^2}} (W\bar{W} + \bar{W}W)$$

The gauge $x_+ = \tau, \quad p_+ = 1$

The kappa-symmetry gauge

DE AZCARRAGA,
LUKIERSKI 1982
SIEGEL, 1983
GREEN, SCHWARZ 1984

Infinitesimal kappa-transformation

$$\chi = \begin{bmatrix} 0 & \theta \\ \eta & 0 \end{bmatrix} \quad \delta\theta = \begin{bmatrix} 0 & 0 & \epsilon_1 & \epsilon_2 & -i\epsilon_2 & -i\epsilon_1 \\ 0 & 0 & \epsilon_3 & \epsilon_4 & -i\epsilon_4 & -i\epsilon_3 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}$$

$f_1(\vartheta) \equiv [\vartheta, \Sigma_+]$ The variation is in the kernel

Gauge equivalence classes labeled by

$$W_F / \text{Ker } f_1 \sim \text{Im } f_1$$

The gauge $\chi = [\Sigma_+, \xi]$

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Supercharges

The supercurrent

$$J^\alpha = g \left(\gamma^{\alpha\beta} A_\beta^{(2)} + \frac{\kappa}{2} \epsilon^{\alpha\beta} (A_\beta^{(3)} - A_\beta^{(1)}) \right) g^{-1}$$

DE WIT, FREEDMAN 1975

Once the kappa gauge has been imposed,
the action of supersymmetry transformations is modified:

$$g \rightarrow e^\epsilon g e^{\tilde{\epsilon}}$$

The supercharges

$$\begin{aligned} Q_\alpha^a = & \frac{i}{4} \int d\sigma e^{-i\frac{x_-}{2}} \left(2p_y \chi_\alpha^a + 2\epsilon^{ab} (Z^*)_b^c (\epsilon_{\alpha\beta} \bar{\chi}_c^\beta + i\epsilon_{cd} \chi_\alpha'^d) - \right. \\ & - 2i\epsilon^{ab} (P_z^*)_b^c \epsilon_{\alpha\beta} \bar{\chi}_c^\beta - i\epsilon_{\alpha\beta} \bar{w}^\beta (\kappa^{a,+1} - 2i(\bar{\kappa}')^{a,-1}) - i\epsilon^{ab} w_\alpha (\kappa_b^{-1} - 2i(\bar{\kappa}')_b^{+1}) + \\ & \left. + 2\epsilon^{ab} P_{w,\alpha} \kappa_b^{-1} + 2\epsilon_{\alpha\beta} \bar{P}_w^\beta \kappa^{a,+1} - 2i y (\chi_\alpha^a + i\epsilon^{ab} \epsilon_{\alpha\beta} (\bar{\chi}')_b^\beta) \right) \end{aligned}$$

x'_- determined from the Virasoro conditions

The central extension

The centrally-extended $\mathfrak{su}(2|2) \oplus \mathfrak{u}(1)$

:

$$[R_\alpha^\beta, R_\gamma^\delta] = \delta_\alpha^\delta R_\gamma^\beta - \delta_\gamma^\beta R_\alpha^\delta$$

$$[L_a^b, L_c^d] = \delta_a^d L_c^b - \delta_c^b L_a^d$$

$$\{Q_\alpha^a, \bar{Q}_b^\beta\} = \delta_\alpha^\beta L_b^a - \delta_b^a R_\alpha^\beta + \frac{1}{2} \delta_b^a \delta_\alpha^\beta H$$

$$\{Q_\alpha^a, Q_\beta^b\} = \epsilon_{\alpha\beta} \epsilon^{ab} P_1$$

$$\{\bar{Q}_a^\alpha, \bar{Q}_b^\beta\} = \epsilon_{ab} \epsilon^{\alpha\beta} P_2.$$

$$P_1 = -\frac{i}{2} \int d\sigma e^{-ix_-} x'_- = \frac{1}{2} e^{-ix_-(-\infty)} (e^{-ip} - 1) = \frac{\xi}{2} (e^{-ip} - 1)$$

$$P_2 = \bar{P}_1$$

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The dispersion relation follows

$$\epsilon(p) = \sqrt{1 + 16h(\lambda)^2 \sin^2\left(\frac{p}{2}\right)}$$

Conclusions and outlook

- Kappa-symmetry gauge which does not break bosonic symmetries of the light-cone gauge
- Central extension as a function of world-sheet momentum
- Loop corrections to the central extension?
- Calculation of the S-matrix?