The $\langle \mathrm{tr}(A_\mu^2) \rangle$ condensate in commutative and noncommutative theories

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Overview of dimension two condensates in gluodynamics

$\langle \mathrm{tr}(A_\mu^2) \rangle$ and $\langle \bar{c}^a c^a \rangle$

Contribute to OPEs, for example,

$$\int d^4x \; e^{ipx} \; (T A_\mu^a(x) A_\nu^b(0)) \underset{p\to\infty}{\longrightarrow} C_{\mu\nu}^{[1]ab}(p)\cdot 1 + C_{\mu\nu}^{[A_\rho^2]ab}(p)\cdot (A_\rho^c)^2 + C_{\mu\nu}^{[\bar{c}c]ab}(p)\cdot \bar{c}^d c^d + \ldots$$

The gluon condensate may be sensitive to various topological defects such as Dirac strings and monopoles.

[F.V.Gubarev, L.Stodolsky, V.I.Zakharov, 2001]

Curci-Ferrari gauge:

$$\mathfrak{O} \equiv \int d^4x \ \left(\mathrm{tr}(A_\mu^2) - \alpha' \bar{c}^a c^a\right)$$

BRST-invariant on-shell

[K.-I. Kondo, 2001]

Landau-type α -gauges:

$$\frac{d}{d\alpha}\langle \int d^4x \ A_\mu^2(x) \rangle|_{\alpha=0} = \langle \bar{c}^a(x) c^a(x) \rangle|_{\alpha=0}$$

[D.B., A.A.Slavnov, 2005]

Gauge theory on a noncommutative plane $\mathbb{R}^4_\theta$

$$[x_\mu, x_\nu] = i\xi\,\theta_{\mu\nu}; \quad \theta^2 = -I.$$

Weyl ordering

Product of operators $\longleftrightarrow$ Moyal product of symbols

$$f(x) * g(x) \overset{\text{def}}{=} e^{i\frac{\xi}{2}\theta_{\mu\nu}\partial^\mu \otimes \partial^\nu} f \otimes g.$$

[Review: M.Douglas, N.Nekrasov, 2001]

Noncommutative gauge theory as a matrix model

Action:

$$S[B] = \int d^4x \ \mathrm{tr}(B_\mu(x) * B_\nu(x) - B_\nu(x) * B_\mu(x) - \xi^{-2}\theta_{\mu\nu})^2$$

$B_\mu(x)$ is a field with values in the $\mathfrak{u}(n)$ Lie algebra

The shift $B_\mu(x) = A_\mu(x) - \xi^{-1}\,\theta_{\mu\nu}\,x^\nu$

leads us to conventional gauge theory with action

$$S[A] = \int d^4x \ \mathrm{tr}(F_{\mu\nu}^2)$$

Gauge transformations in the matrix model language

$$B_\mu^\omega(x) = \omega(x) * B_\mu(x) * \omega^\dagger(x)$$

(homogeneous!)

Thus, we have a gauge-invariant (non-local) operator

$$\int d^4x \ \mathrm{tr}(B_\mu * B^\mu)$$

In the conventional gauge theory language this operator is

$$\int d^4x \;\; \mathrm{tr}(A_\mu * A^\mu - 2\xi^{-1} A_\mu \theta^{\mu\nu} x_\nu + \xi^{-2} x^2)$$

[A.A.Slavnov, 2004]

an (infinite) constant

operator with zero v.e.v.: $\langle A_\mu \rangle = 0$

operator with the desired condensate as its v.e.v.

Is the condensate $\langle \mathrm{tr}(A_\mu^2) \rangle$ gauge-invariant?

Important question arises:

$$\int d\Omega \, \mathrm{tr}(A * B - B * A) = 0 \qquad ?$$

This is not always true !

Counterexample – the partial isometry operator $S = P \cdot U$

$$SS^{\dagger}S = S$$

$$S = \sum_{n=0}^{\infty} |n\rangle\langle n+1|$$

$$\Rightarrow SS^{\dagger} = 1; \; S^{\dagger}S = 1 - |0\rangle\langle 0|$$

$$\mathrm{tr}([S, S^{\dagger}]) = 1$$

Back to our case

[R.N.Baranov, D.B., A.A.Slavnov, 2006]

Under a «quantum» gauge transformation

$$\delta A_\mu = \frac{\delta\alpha}{2\alpha} D_\mu \int M^{-1}(x,y)\,\partial A(y)\,dy \qquad M = \partial_\mu \nabla^\mu$$

the variation of the condensate is **non-zero**:

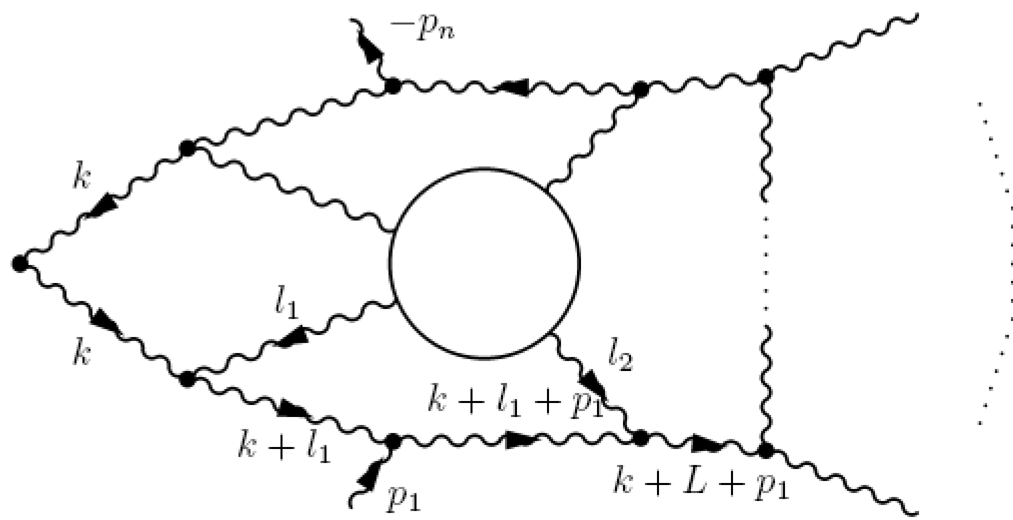
$$\delta\langle C\rangle = -\delta\alpha\, D_c(0) \neq 0$$

Here $\omega(x) = \exp\left(\frac{\delta\alpha}{2\alpha}\int M^{-1}(x,y)\,\partial A(y)\,dy\right)$ is fixed!

1. The operator is still gauge-invariant, if one considers Ward identities.

2. The volume-averaged operator decouples from Green's functions

$$\lim_{\Omega\to\infty}\frac{1}{\Omega}\int_{\Omega} d^4x\,\langle [A_\mu^2(x)]\cdot\prod_i O_i(y_i)\rangle = \langle [A_\mu^2(0)]\rangle\cdot\langle\prod_i O_i(y_i)\rangle$$



Result:

The condensate is invariant under a limited set of gauge transformations.

Thank you for attention.